

# $k$ DFAs or MDFAs: What to choose

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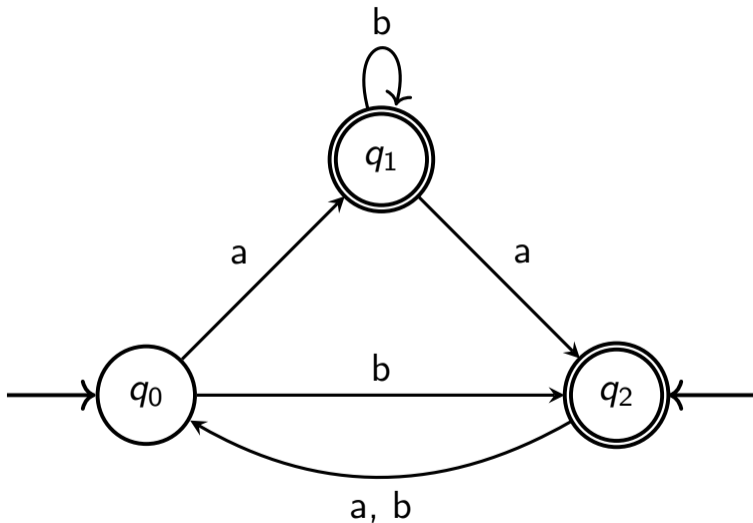
# DFA vs $k$ DFA vs MDFA

- **Standard DFA:** One initial state ( $q_1$ ).
- **$k$ DFA:** A set of initial states  $I = \{i_1, \dots, i_k\}$  ( $k \geq 1$ ).
- **MDFA:** A set of initial states  $I, 1 \leq |I| \leq n$ .
- **Nondeterminism?** Only at the start, the transitions remain deterministic.
- **The Language:**

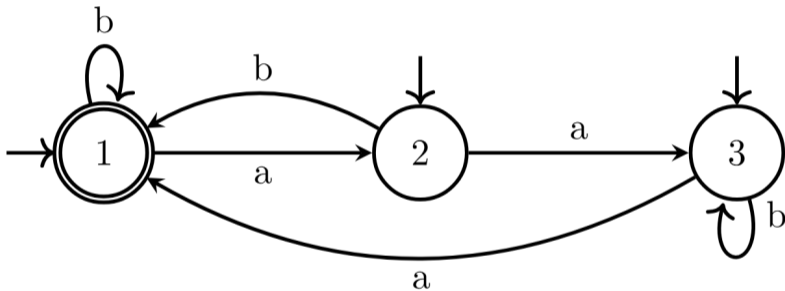
$$L(M) = \bigcup_{i \in I} L(M_i)$$

It accepts the *union* of languages starting from each initial point.

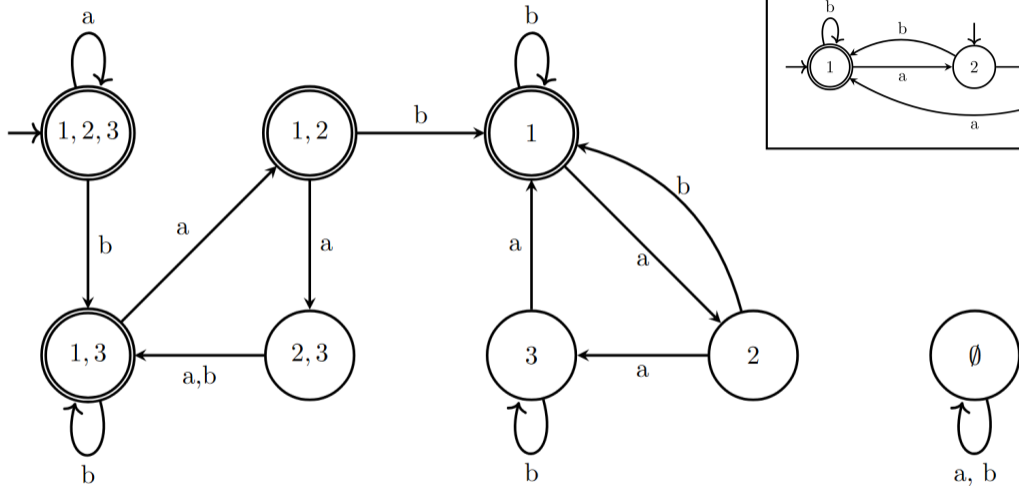
# $k$ DFA/MDFA example



# Example



# Example



# Magic numbers problem

## Magic numbers problem

The question whether there exists a  $n$ -state  $k$ DFA whose equivalent minimal DFA has  $\alpha$  states, for all  $n, \alpha \in \mathbb{N}^+$  satisfying  $\alpha < 2^n$ .

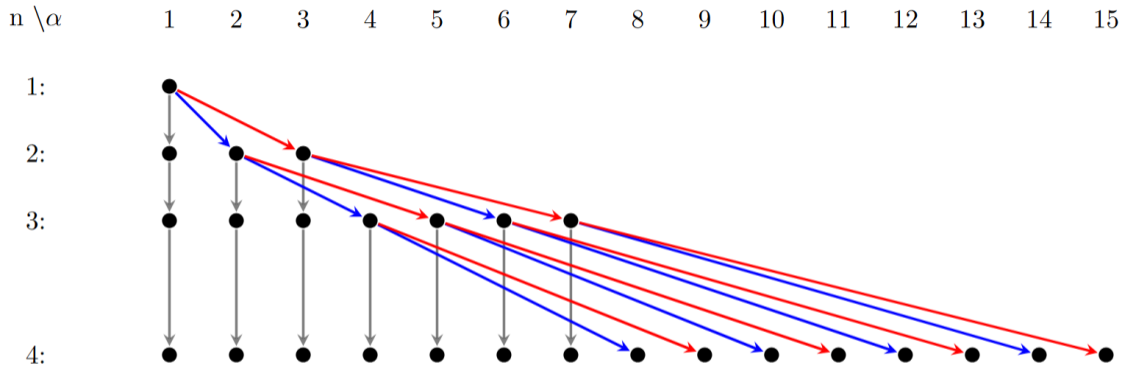
A number  $\alpha$  not satisfying this condition is called a magic number (for  $n$ ).

## Theorem 3.3

For every pair  $(n, \alpha)$ , where  $n, \alpha \in \mathbb{N}^+$  such that  $\alpha \leq 2^n - 1$ , there exists an  $n$ -state  $k$ DFA whose equivalent minimal DFA has exactly  $\alpha$  states.

Moreover, the automata are over an alphabet of size at most  $2n$ .

# Magic numbers problem - range



Construction	Added	Effect
C1 (gray)	state $n$	$(n, \alpha) \rightarrow (n+1, \alpha)$
C2 (blue)	state $n$ , letter $a_n$	$(n, \alpha) \rightarrow (n+1, 2\alpha)$
C3 (red)	state $n$ , letters $a_n, b_n$	$(n, \alpha) \rightarrow (n+1, 2\alpha+1)$

# Average state complexity

## Theorem 4.1

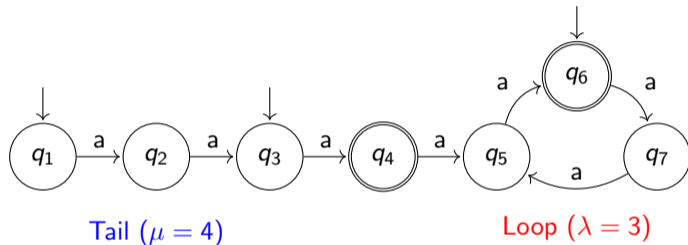
Average state complexity of a language represented by an  $n$ -state  $k$ DFA is at most

$$\frac{\sum_{i=1}^n \left( \binom{n}{i} \cdot \sum_{j=1}^i \binom{n}{j} \right)}{\sum_{i=1}^n \binom{n}{i}}$$

## Theorem 4.3

Let  $n \in \mathbb{N}^+$  then average state complexity of a language represented by an  $n$ -state family automaton is at most  $5/8 \times 2^n$ .

# Unary $k$ DFA/MDFA

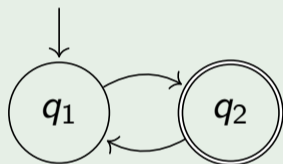


Here,  $n = 7$ . We have multiple entry points into a deterministic structure.

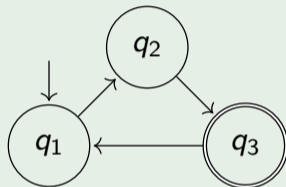
## Problem 1: The Multi-Union

Since  $L(M) = \bigcup L(M_i)$ , finding the minimal DFA is equivalent to finding the union of  $m$  unary languages.

### Example



a, aaa, aaaaa, ...



aa, aaaaa, aaaaaaaa, ...

**Resulting Complexity:** The equivalent minimal DFA must track *both* periods.

$$\text{New Loop Size} = \text{lcm}(\lambda_1, \lambda_2) = \text{lcm}(2, 3) = 6$$

The general bound is:

$$\text{Size} \approx \text{lcm}(\lambda_1, \dots, \lambda_m) + \max(\mu_1, \dots, \mu_m)$$

## Problem 2: Conversion of $k$ DFA to DFA

What if the graph is connected (one component)?

- **Intuition:** All initial states feed into the *same* cycle.
- Starting later in the tail just shifts the language:  $L(q_j) = a^{-j}L(q_1)$ .
- The period  $\lambda$  does not change

### Upper Bound

For a weakly connected  $n$ -state  $k$ DFA, the equivalent minimal DFA has **at most  $n$  states**.

**Why?** We don't need to multiply loop lengths. We just overlay the shifted languages on the same structure.

## Problem 2: Conversion of $k$ DFA to DFA

If the  $k$ DFA has multiple disconnected components:

- We can pick initial states in totally different loops.
- This triggers the LCM explosion we saw earlier.

### Upper Bound

For components with sizes  $(\lambda_i, \mu_i)$ :

$$\text{States} \leq \text{lcm}(\lambda_1, \dots, \lambda_m) + \max(\mu_1, \dots, \mu_m)$$

This is related to the **Landau Function**  $g(n)$ , the maximal order of an element in the symmetric group  $S_n$ .

$$g(n) = \max\{\text{lcm}(x_1, \dots, x_m) \mid x_1 + \dots + x_m = n\} = e^{\sqrt{n \ln n}(1+o(1))}$$

## Problem 3: Magic Numbers

Can we always build a minimal DFA of size exactly  $\alpha$  from an  $n$ -state  $k$ -entry DFA?

### Weakly Connected:

- Yes.
- We can achieve any size  $\alpha \in [1, n]$ .
- **Strategy:** Pick initial states to "skip" parts of the tail.

### General Case:

- No.
- Gaps appear near the Landau function limit.
- **Example:** We cannot reach  $g(n) - 1$  states for  $n \geq 7$ .

# Future: Automata and AI?

- State complexity prediction using machine learning methods
- Automated extraction of automata from images

# Thank You!

*Any Questions?*