k-entry DFA

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k-entry deterministic finite-state automaton $({}_{k}DFA)$ is a quintuple $M = (Q, \Sigma, I, F, \delta)$ where

- $\bullet~Q$ is a finite set of states
- Σ is a finite set of input symbols
- I is a set of initial states, $I \subseteq Q$
- σ is a transition function, $\sigma:Q\times\Sigma\to Q$
- $\bullet~F$ is a set of final states, $F\subseteq Q$

Goals

- 1. Develop a program that accepts an automaton as input and generates all automata with various choices of initial states. Expand this program to generate all n-state automata. Additionally, create a program capable of determinizing and minimizing the automaton to ascertain the state complexity of the language it represents. Furthermore, ensure that the program is designed to leverage parallel computing.
- 2. Investigate the deterministic state complexity of automata represented by nondeterministic automata, where the only nondeterminism is from a choice of initial states.
- 3. Examine the worst-case state complexity identified in 2.
- 4. Explore the range of all obtainable state complexities from 2.
- 5. Study the average state complexity from 2.

Generation



Generation



Generation



Theorem 3.2 Let $n \in \mathbb{N}$, $n \ge 1$, then average state complexity of a language represented by an *n*-state family is at most $5/8 \times 2^n$.

$$\frac{\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{i}\binom{n}{i}\binom{n}{j}}{2^{n}-1} \leq \frac{5}{8}2^{n}$$

$$S = \sum_{i=1}^{n} \sum_{j=1}^{i} \binom{n}{i} \binom{n}{j} = \sum_{1 \le j \le i \le n} \binom{n}{i} \binom{n}{j} = \sum_{1 \le i \le j \le n} \binom{n}{i} \binom{n}{j}$$

$$2S = \sum_{1 \le i \le j \le n} {\binom{n}{i} \binom{n}{j}} + \sum_{1 \le j \le i \le n} {\binom{n}{i} \binom{n}{j}} =$$

$$= \sum_{1 \le i \le j \le n} {\binom{n}{i} \binom{n}{j}} + \sum_{1 \le j < i \le n} {\binom{n}{i} \binom{n}{j}} + \sum_{1 \le i = j \le n} {\binom{n}{i} \binom{n}{j}} =$$

$$= \sum_{1 \le i, j \le n} {\binom{n}{i} \binom{n}{j}} + \sum_{i = 1}^{n} {\binom{n}{i}}^{2} =$$

$$= (2^{n} - 1)^{2} + {\binom{2n}{n}} - 1.$$

$$S = \frac{(2^{n} - 1)^{2} + {\binom{2n}{n}} - 1}{2}.$$

$$\frac{(2^n - 1)^2 + \binom{2n}{n} - 1}{2(2^n - 1)} \le \frac{5}{8}2^n$$

$$2^{2n} - 2 \cdot 2^n + 1 + \binom{2n}{n} - 1 \le \frac{5}{4} 2^n (2^n - 1)$$
$$\binom{2n}{n} \le \frac{4^n}{4} + \frac{3 \cdot 2^n}{4}.$$

Worst-case state complexity

Lemma 2.1 For every $n \in \mathbb{N}$ there exists a *n*-state $_kDFA$ such that equivalent minimal DFA has exactly $2^n - 1$ states.



Worst-case state complexity



n-state $_k$ DFA $\rightarrow \alpha$ -state DFA

for every pair (n, α) , $n, \alpha \in \mathbb{N}^+ \alpha \leq 2^n - 1$







Magic numbers - unary





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Thank you!