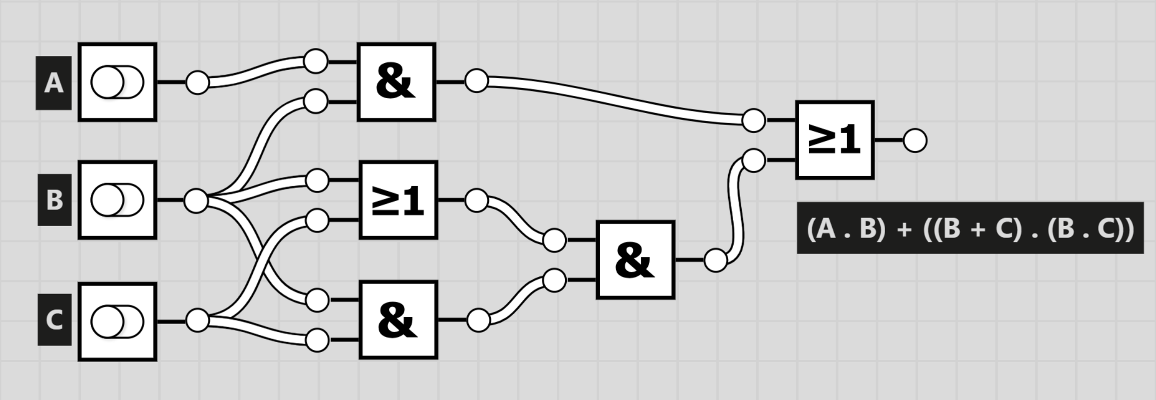
Partially nondeterministic automata - Nondeterministic choice of initial states

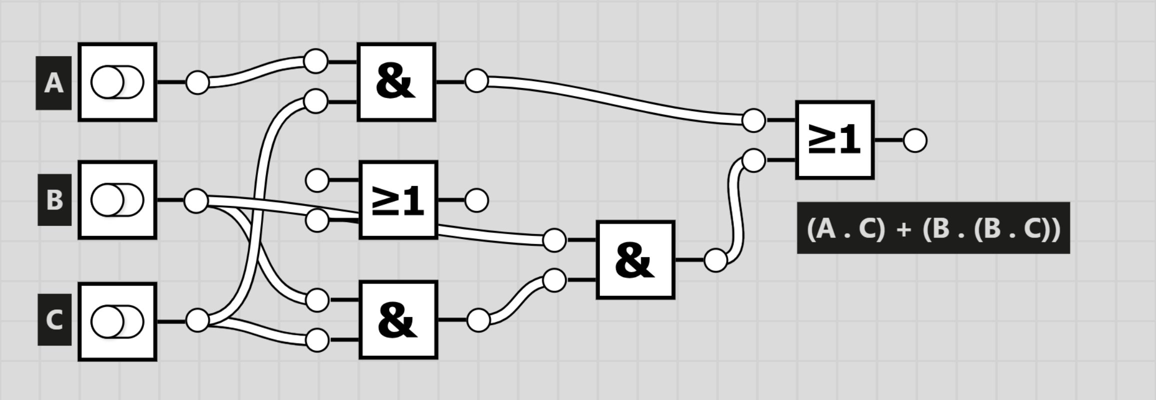
Bc. Šimon Huraj
Tutor: RNDr. Juraj Šebej, PhD.

Master's thesis defense
May 27, 2025

Motivation



Motivation



DFA vs ${}_k$DFA

Definition[1]

Deterministic finite-state automaton DFA is a quintuple $D = (Q, \Sigma, \delta, i, F)$ where

- Q is a finite set of states
- Σ is a finite set of input symbols
- σ is a transition function, $\sigma : Q \times \Sigma \to Q$
- i is a initial state, $i \in Q$
- F is a set of final states, $F \subseteq Q$

Definition[2]

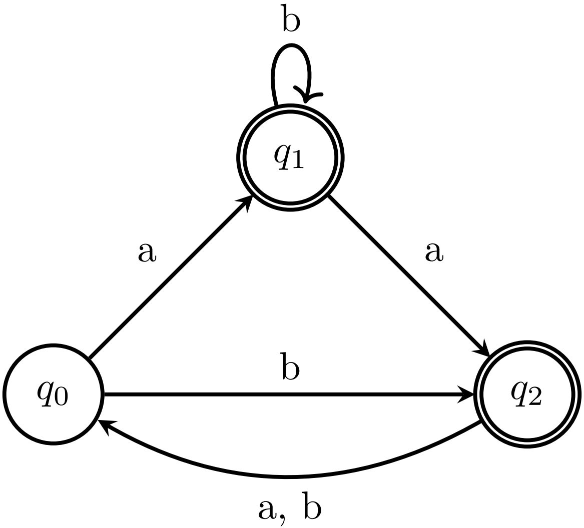
k-entry deterministic finite-state automaton ${}_k$DFA is a quintuple $M = (Q, \Sigma, \delta, I, F)$ where

- Q is a finite set of states
- Σ is a finite set of input symbols
- σ is a transition function, $\sigma : Q \times \Sigma \to Q$
- **I is a set of initial states, $I \subseteq Q$**
- F is a set of final states, $F \subseteq Q$

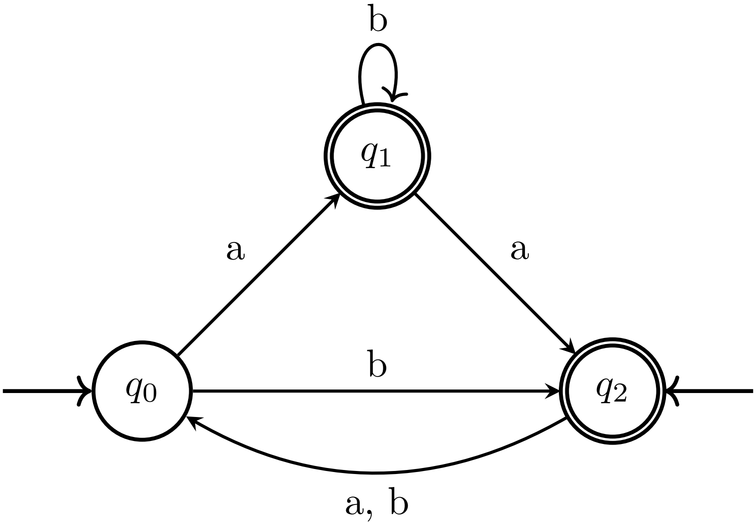
[1] (J. E. Hopcroft, R. Motwani, and J. D. Ullman. *Introduction to automata theory, languages, and computation.* Vol. 2. Addison-Wesley, 2003. ISBN: 978-0201441246)

[2] (M. Holzer, K. Salomaa, and S. Yu. "On the State Complexity of k-Entry Deterministic Finite Automata". In: *Journal of Automata, Languages and Combinatorics* 6.4 [2001], pp. 453–466)

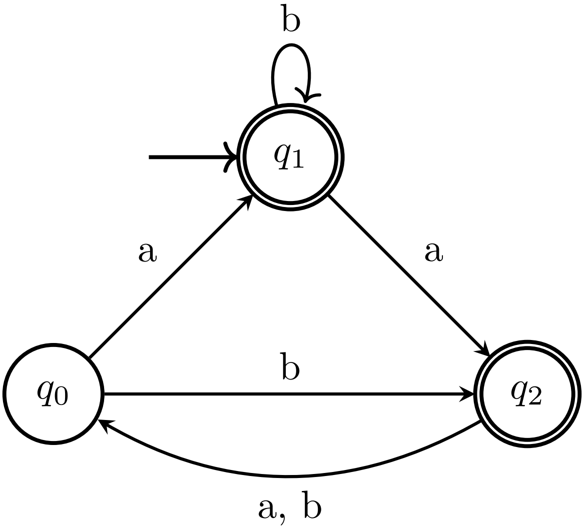
${}_k$DFA example



${}_k$DFA example



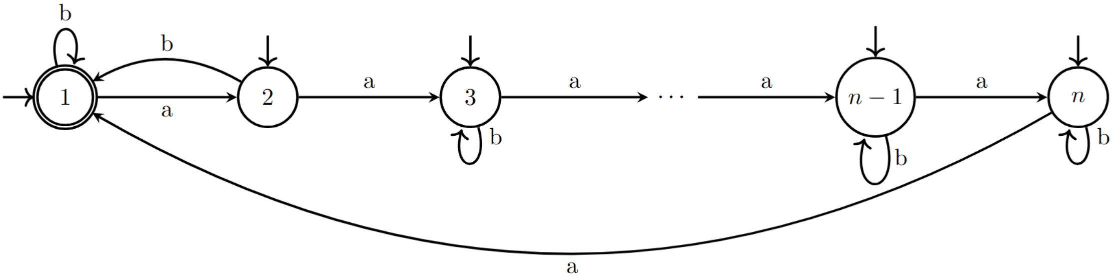
${}_k$DFA example



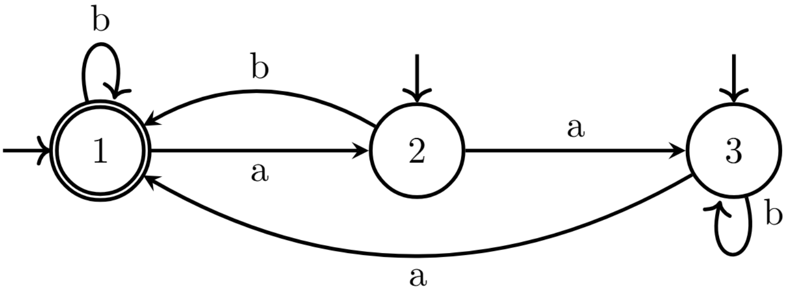
Conversion state complexity - upper bound

Lemma 3.1

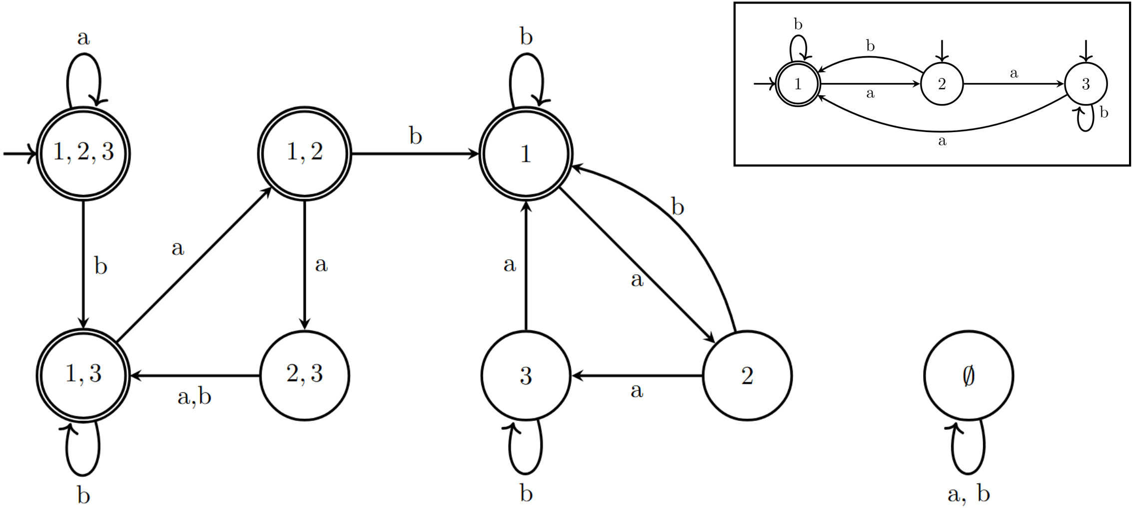
For every $n \in \mathbb{N}^+$ there exists a n-state ${}_k$DFA such that equivalent minimal DFA has exactly $2^n - 1$ states.



Example



Example

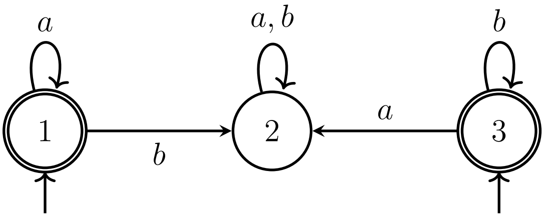


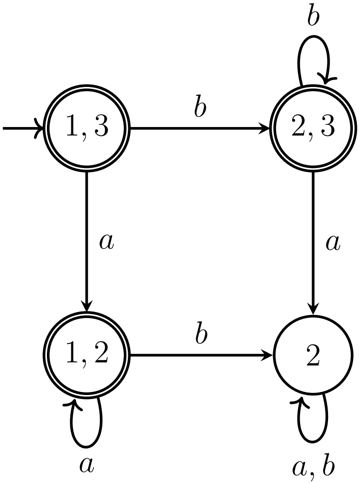
Conversion state complexity

3-state ${}_k$DFA $\rightarrow$ 4-state DFA ?

Conversion state complexity

3-state ${}_k$DFA $\rightarrow$ 4-state DFA ?





Magic numbers problem

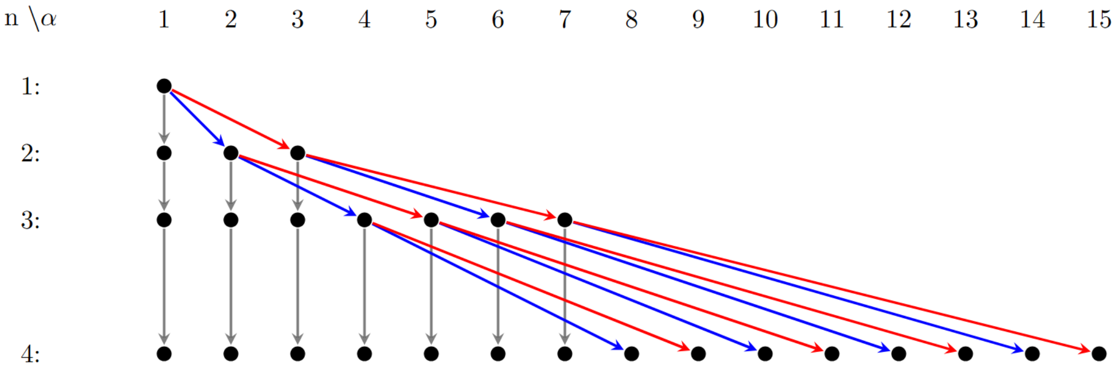
Magic numbers problem

The question whether there exists a n-state ${}_k$DFA whose equivalent minimal DFA has α states, for all $n, \alpha \in N^+$ satisfying $\alpha < 2^n$.
A number α not satisfying this condition is called a magic number (for n).

Theorem 3.3

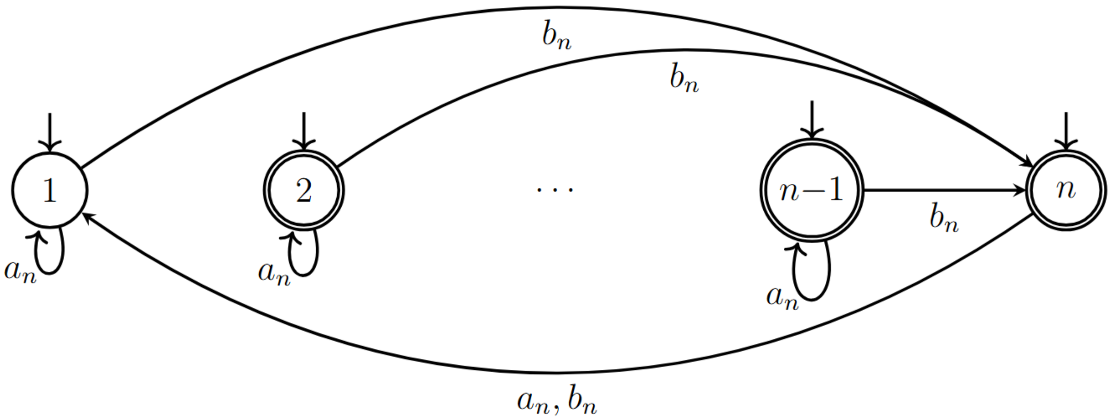
For every pair (n, α), where $n, \alpha \in \mathbb{N}^+$ such that $\alpha \leq 2^n - 1$, there exists an n-state ${}_k$DFA whose equivalent minimal DFA has exactly α states.
Moreover, the automata are over an alphabet of size at most $2n$.

Magic numbers problem - range



Construction	Added	Effect
C1 (gray)	state n	$(n, \alpha) \to (n+1, \alpha)$
C2 (blue)	state n, letter a_n	$(n, \alpha) \to (n+1, 2\alpha)$
C3 (red)	state n, letters a_n, b_n	$(n, \alpha) \to (n+1, 2\alpha+1)$

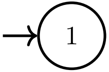
Magic numbers problem - range



Magic numbers problem - range - example

4-state ${}_k$DFA $\rightarrow$ 6-state DFA

n = 1: 1



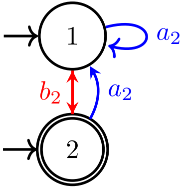
Magic numbers problem - range - example

4-state ${}_k$DFA $\rightarrow$ 6-state DFA

n = 1: 1

$\searrow$ C3

n = 2: 1 2 3



Magic numbers problem - range - example

4-state ${}_k$DFA $\rightarrow$ 6-state DFA

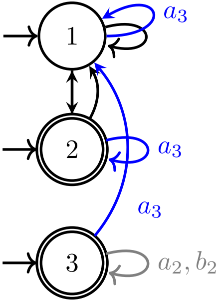
n = 1: 1

$\searrow$ C3

n = 2: 1 2 3

$\downarrow$ C2

n = 3: 1 $\cdots$ 6 7



Magic numbers problem - range - example

4-state ${}_k$DFA $\rightarrow$ 6-state DFA

n = 1: 1

$\searrow$ C3

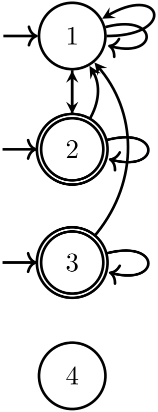
n = 2: 1 2 3

$\downarrow$ C2

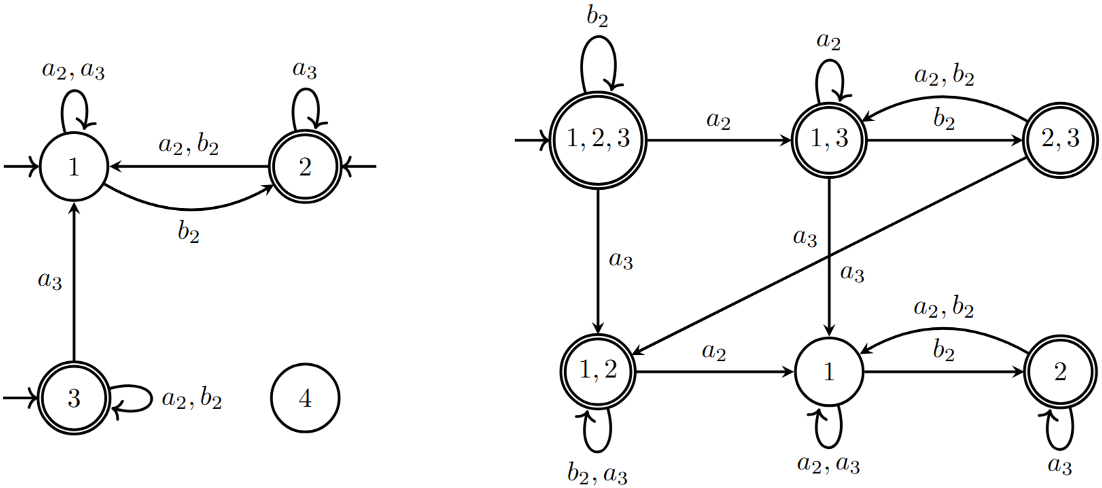
n = 3: 1 $\cdots$ 6 7

$\downarrow$ C1

n = 4: 1 $\cdots$ 6 $\cdots$ 15



Magic numbers problem - range - example



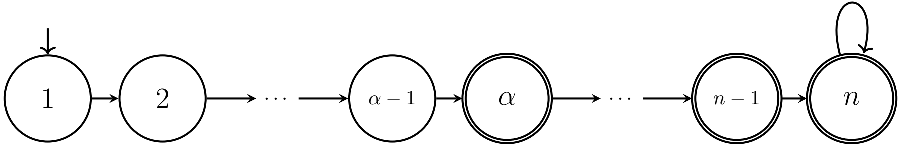
Magic numbers problem - unary automata

Theorem 3.4

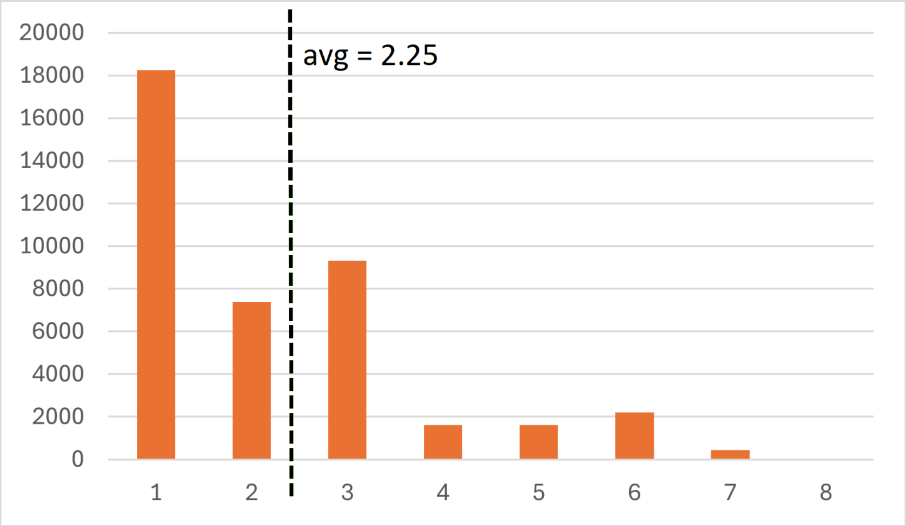
Let M be an n-state ${}_k$DFA over a unary alphabet. Then, the equivalent minimal DFA has at most n states.
Moreover, for every pair (n, α), where $n, \alpha \in \mathbb{N}^+$, there exists a unary n-state ${}_k$DFA such that its equivalent minimal DFA has exactly α states if and only if $\alpha \leq n$.

Lemma 3.7

For every pair (n, α), where $n, \alpha \in \mathbb{N}^+$ such that $\alpha \leq n$, there exists a unary n-state ${}_k$DFA whose equivalent minimal DFA has exactly α states.



Average state complexity



Average state complexity

Theorem 4.1[3]

Average state complexity of a language represented by an n-state ${}_k$DFA is at most

$$\frac{\sum_{i=1}^{n}\left(\binom{n}{i}\cdot\sum_{j=1}^{i}\binom{n}{j}\right)}{\sum_{i=1}^{n}\binom{n}{i}}$$

Theorem 4.3

Let $n \in \mathbb{N}^+$ then average state complexity of a language represented by an n-state family automaton is at most $5/8 \times 2^n$.

[3] (Š. Huraj. *Partially nondeterministic automata - Nondeterministic choice of initial states*. Bachelor's thesis, P. J. Safarik University. 2023)

Average state complexity

n	Average state complexity obtained by Theorem 4.1	Average state complexity obtained by Theorem 4.3	Actual average state complexity	2^n
2	2.34	2.5	1.39	4
3	4.86	5	2.25	8
4	9.80	10	3.81	16
5	19.55	20	6.37	32
6	38.83	40	10.24	64
7	77.01	80	15.81	128

Note: Alphabet of size 2 (computations).
Does not depend on alphabet size (formulas).

Summary and future work

- ${}_kDFA$ is a DFA except every state can be initial, similar as electrical (logical) circuit we can start it with different configurations
- computational results - average s.c., distinct languages, number of distinct s.c., bigger alphabets
- formal result - magic numbers problem with linear alphabet, unary magic numbers problem, average state complexity

Thank you for your attention

Questions

Question 1

Pre konverziu ${}_k$DFA na DFA bolo dokázané horné ohraničenie $\frac{5}{8}2^n$ pre priemerný počet stavov. Dá sa niečo povedať o tom, ako sa toto horné ohraničenie zmení, ak je počet počiatočných stavov fixovaný, t.j., pre fixovanú hodnotu k ?

Questions

Question 2

Strana 47, Lema 3.7: v čom je prínos tejto lemy oproti predošlej Vete 3.4 zo strany 44 ?